PONTRYAGIN'S MAXIMUM PRINCIPLE AND OPTIMIZATION OF THE FLIGHT PHASE IN SKI JUMPING

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There are several factors (the initial ski jumper's body position and its changes at the transition to the flight phase, the magnitude and the direction of the velocity vector of the jumper's center of mass, the magnitude of the aerodynamic drag and lift forces, etc.), which determine the trajectory of the jumper ski system along with the total distance of the jump. The objective of this paper is to bring out a method based on Pontryagin's maximum principle, which allows us to obtain a solution of the optimization problem for flight style control with three constrained control variables – the angle of attack (α), body ski angle (β), and ski opening angle (V). The flight distance was used as the optimality criterion. The borrowed regression function was taken as the source of information about the dependence of the drag (D) and lift (L) area on control variables with tabulated regression coefficients. The trajectories of the reference and optimized jumps were compared with the K = 125 m jumping hill profile in Frenštát pod Radhoštěm (Czech Republic) and the appropriate lengths of the jumps, aerodynamic drag and lift forces, magnitudes of the ski jumper system's center of mass velocity vector and it's vertical and horizontal components were evaluated. Admissible control variables were taken at each time from the bounded set to respect the realistic posture of the ski jumper system in flight. It was found that a ski jumper should, within the bounded set of admissible control variables, minimize the angles α and β, whereas angle V should be maximized. The length increment due to optimization is 17%. For future work it is necessary to determine the dependence of the aerodynamic forces acting on the ski jumper system on the flight via regression analysis of the experimental data as well as the application of the control variables related to the ski jumper's mental and motor abilities.

Keywords: Computer simulation, ski jumper, optimal control, aerodynamic force.

INTRODUCTION

Aerodynamic drag and lift forces, the initial movement state of the flight phase of the ski jump, together with the gravitational force, determine the trajectory of the ski jumper system's center of mass along with the total distance of the jump. The mentioned forces are substantially influenced by the skier's course of posture (Jin, Shimizu, Watanuki, Kubota, & Kobayashi, 1995; Müller, Platzer, & Schmölzer, 1996; Schmölzer & Müller, 2002; Schmölzer & Müller, 2005; Virmavirta et al., 2005), the aerodynamic qualities of the ski jumper's equipment (Meile et al., 2006) and his/her somatotype (Vaverka, 1994; Schmölzer & Müller, 2005; Müller, W., Gröschl, Müller, R., & Sudi, 2006).

Computer simulations, among other things, can help to clarify the optimization strategy of the flight style. There is only one pattern of the optimal change in the angle of attack, thus affording maximal flight length (Remizov, 1984). To solve the optimization problem with one control variable, Remizov applied Pontryagin's maximum principle (Pontryagin, 1962) and computations were based on data from wind tunnel experiments (Grozin, 1971). It was shown that the angle of attack should gradually increase according to a convex function. A solution of the optimization problem of the flight phase in ski jumping demands data describing the dependency of the aerodynamic forces on angle parameters of the flight style. Seo and Murakami (2003) took one control variable (the forward leaning angle) to solve the optimization problem. Their result showed that a jumper should keep the forward leaning angle of the magnitude of 6°. As a new style named the V-style was extended in the 1990's, so far a sufficient amount of information about the contemporary range of the flight position and appropriate aerodynamic force acting on the ski jumper system and the jumper's equipment has not been found. Thus, Seo, Watanabe and Murakami (2004) made wind tunnel experiments with full size models to acquire data for sufficiently wide ranges of angles of attack, body ski angles and ski opening angles. Seo, Murakami and Yoshida (2004) solved the optimization problem by using two control variables: the body ski angle and the ski opening angle. The authors used data prepared by Seo, Watanabe and Murakami (2004).

Meile et al. (2006) investigated the aerodynamic behaviour of a ski jumper model in a reduced scale (1:√2) in a wide range of angles of attack. The experimental results were in good agreement with full scale measurements on athletes. Murakami, Hirai, Seo and Ohgi (2008) derived aerodynamic forces from the data analysis of a high speed video image of the initial flight phase. They concluded that the aerodynamic forces, which were extracted from the image, were in reasonable agreement with existing wind tunnel data for the cases of jumping flights in the quasi steady flight phase.

Hermsdorf, Hildebrand, Hofmann and Müller (2008) introduced a biomechanical multibody model for simulation in ski jumping that makes possible the evaluation of the time course of joint angles, global position and the orientation of the ski jumper.

The simple ski jumping model for the calculation of jump length was evaluated by Schindelwig and Nachbauer (2007). Wind velocity plays an important role in the agreement between the measured and calculated jump lengths.

The aim of this study is to bring out a method for obtaining a solution to the optimization problem for flight style control with three control variables (angle of attack, body ski angle, ski opening angle) and correction of the concrete flight style based on the solution of the optimization problem. Admissible control variables were taken from the bounded set to respect the realistic posture of the ski jumper system in flight. The influence of the wind was neglected.

METHODS

Formulation of the optimization problem and the methodology used in the numerical computation of the optimal flight style

A ski jumper controls, over time, flight position in order to maximize flight distance and securely finish a jump by landing and outrunning. The way to find the optimal control of flight position was a matter of solving an optimization problem. An element of the optimization problem formulation for a ski jumping flight is the mathematical modeling of the movement of the center of mass of a ski jumper system. Consider the jumper moving through still air on a vertical plane. For the simulation made here the coordinate system O_{xy} with the horizontal x axis oriented forward in the direction of the flight, the vertical y axis oriented downward and the origin blending with the center of mass of the ski jumper system above a ski jump edge have been used. A ski jumper controls body segment angles, the angle of attack and the ski opening angle.

A ski jumper system's center of mass and state of motion are determined by the coordinates x, y and velocity vector components v_x and v_y in the coordinate system O_{xy}. Thus, the vector of the state variables $\boldsymbol{s}$ takes form:

$$\boldsymbol{s} = (v_x(t), v_y(t), x(t), y(t)). \tag{1}$$

The equation of motion $m\boldsymbol{a} = \boldsymbol{F}_g + \boldsymbol{F}_D + \boldsymbol{F}_L$, where $m\boldsymbol{a}$ is the net force, is rearranged to obtain the components of the acceleration and the velocity vector

$$\begin{aligned}
\frac{dv_x}{dt} &= -\frac{\rho D(\boldsymbol{u}(t))}{2m} v_x \sqrt{v_x^2 + v_y^2} + \frac{\rho L(\boldsymbol{u}(t))}{2m} v_y \sqrt{v_x^2 + v_y^2} \\
\frac{dv_y}{dt} &= g - \frac{\rho D(\boldsymbol{u}(t))}{2m} v_y \sqrt{v_x^2 + v_y^2} - \frac{\rho L(\boldsymbol{u}(t))}{2m} v_x \sqrt{v_x^2 + v_y^2} \\
\frac{dx}{dt} &= v_x \\
\frac{dy}{dt} &= v_y
\end{aligned} \tag{2}$$

and these equations model the state of motion progress in time and take into account forces acting on the ski jumper system in the inertial system of coordinates O_{xy} – gravitation $\boldsymbol{F}_g = m\boldsymbol{g}$ (m being the mass and $\boldsymbol{g}$ the gravitational acceleration), aerodynamic drag $\boldsymbol{F}_D = \frac{1}{2}\rho C_D S_f v^2 \boldsymbol{i} = \frac{1}{2}\rho D v^2 \boldsymbol{i}$ (ρ being the air density, C_D the drag coefficient, S_f the frontal area, D the drag area, $v = \sqrt{v_x^2 + v_y^2}$ the magnitude of the velocity vector and $\boldsymbol{i}$ the unit vector in the same direction as the velocity vector) and lift forces $\boldsymbol{F}_L = \frac{1}{2}\rho C_L S_p v^2 \boldsymbol{j} = \frac{1}{2}\rho L v^2 \boldsymbol{j}$ (C_L being the lift coefficient, S_p the planform area, L the lift area and $\boldsymbol{j}$ the unit vector normal to the air stream). Express (2) as:

$$\frac{d\boldsymbol{s}}{dt} = \boldsymbol{f}(\boldsymbol{s}, \boldsymbol{u}). \tag{3}$$

The acquired solution of the optimization problem has to respect the athlete's psychological and physical individuality, which are exhibited in individual flight style. That is the reason why the control vector of the system is assumed to be constrained to belong to a suggested closed and bounded set U^r in r-dimensional space. In all steps of the algorithm consider

$$\forall t \in \langle 0, T \rangle: \ \boldsymbol{u} \in U^r \subset R^r. \tag{4}$$

The flight time T corresponds to the time interval from the start of the reference jump to the instant of the landing of the jumper's model at the intersection of the flight path with a smooth curve S representing the projection of the landing area of the ski jump on a vertical plane containing the coordinate system O_{xy}. The equation of curve S is given as follows:

$$S:\ \psi(x,y)=0\,. \tag{5}$$

Assume for coordinates of the center of mass in the instant T that the following is valid:

$$\psi\big(x(T),y(T)\big)=0\,. \tag{6}$$

To solve the optimization problem, Pontryagin's maximum principle has been applied. The jumper ski system, from the dynamics point of view, can be characterised in terms of a set of first order differential equations (2) in which control variables are to be selected over time to obtain some desirable objectives in an optimal manner. Pontryagin's maximum principle consists of a set of necessary conditions, which must be satisfied by optimal solution and originate in classical calculus of variations (Pierre, 1969, 478).

The functional J is to be minimized by the appropriate selection of $\boldsymbol{u}(t)\in U^r$ at each instant

$$t\in\left\langle 0,T^*\right\rangle : J^*=\min_{\boldsymbol{u}(t)} F\big(\boldsymbol{s}(T),\boldsymbol{u}(T)\big)\,. \tag{7}$$

Because the purpose is to maximize jump length, the functional J has the form

$$J=-x(T)\,. \tag{8}$$

To minimize J, it is necessary to formulate a Hamiltonian that has the following general form (Víteček, 2002; Pierre, 1969; Lewis & Syrmos, 1995):

$$H(\boldsymbol{s},\boldsymbol{u},\boldsymbol{p})=\boldsymbol{p}^{\mathrm{T}}\boldsymbol{f}(\boldsymbol{s},\boldsymbol{u}), \tag{9}$$

where $\boldsymbol{p}(t)$ is the costate vector. On the basis of (2) is

$$\begin{aligned} H=&-p_1K_1(\boldsymbol{u})v_x\sqrt{v_x^2+v_y^2}+p_1K_2(\boldsymbol{u})v_y\sqrt{v_x^2+v_y^2}+p_2g-\\ &-p_2K_1(\boldsymbol{u})v_y\sqrt{v_x^2+v_y^2}-p_2K_2(\boldsymbol{u})v_x\sqrt{v_x^2+v_y^2}+p_3v_x+p_4v_y, \end{aligned} \tag{10}$$

where

$$K_1\big(\boldsymbol{u}(t)\big)=\frac{\rho D}{2m} \text{ or } K_2\big(\boldsymbol{u}(t)\big)=\frac{\rho L}{2m}\,. \tag{11}$$

According to the Pontryagin's maximum principle, the costate vector $\boldsymbol{p}$ must satisfy the system of equations canonically conjugated to the system (2)

$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t}=-\frac{\partial H}{\partial \boldsymbol{s}} \tag{12}$$

and the control vector $\boldsymbol{u}^*(t)$ which leads to the minimum of J is the vector which minimizes Hamiltonian H:

$$H^*\big(\boldsymbol{s}^*,\boldsymbol{u}^*,\boldsymbol{p}^*\big)=\min_{\boldsymbol{u}(t)\in U^r} H\big(\boldsymbol{s}^*,\boldsymbol{u},\boldsymbol{p}^*\big)\,. \tag{13}$$

Our conjugate system for components of the costate vector is obtained on the basis of the equation (10):

$$\begin{aligned} \frac{\mathrm{d}p_1}{\mathrm{d}t}&=-\frac{\partial H}{\partial v_x}=\frac{v_xv_y}{\sqrt{v_x^2+v_y^2}}\big(p_2K_1-p_1K_2\big)-p_3+\\ &+\sqrt{v_x^2+v_y^2}\,\big(p_1K_1+p_2K_2\big)+\frac{v_x^2}{\sqrt{v_x^2+v_y^2}}\big(p_1K_1+p_2K_2\big)\\ \frac{\mathrm{d}p_2}{\mathrm{d}t}&=-\frac{\partial H}{\partial v_y}=\frac{v_xv_y}{\sqrt{v_x^2+v_y^2}}\big(p_1K_1+p_2K_2\big)-p_4+\\ &+\sqrt{v_x^2+v_y^2}\,\big(p_2K_1-p_1K_2\big)+\frac{v_y^2}{\sqrt{v_x^2+v_y^2}}\big(p_2K_1-p_1K_2\big)\\ \frac{\mathrm{d}p_3}{\mathrm{d}t}&=-\frac{\partial H}{\partial x}=0\\ \frac{\mathrm{d}p_4}{\mathrm{d}t}&=-\frac{\partial H}{\partial y}=0. \end{aligned} \tag{14}$$

A corresponding boundary condition is formed (Lewis & Syrmos, 1995):

$$\left(\frac{\partial F}{\partial \boldsymbol{s}}+\frac{\partial \psi}{\partial \boldsymbol{s}}\nu-\boldsymbol{p}\right)\Bigg|_T \mathrm{d}\boldsymbol{s}(T)+\left(\frac{\partial F}{\partial t}+\frac{\partial \psi}{\partial t}\nu+H\right)\Bigg|_T \mathrm{d}T=0\,. \tag{15}$$

Based on comparing (7) and (8) $F\big(\boldsymbol{s}(T)\big)=-x(T)$, so

$$\frac{\partial F}{\partial \boldsymbol{s}}=\begin{bmatrix}0\\0\\-1\\0\end{bmatrix} \tag{16}$$

Also

$$\frac{\partial \psi}{\partial \boldsymbol{s}}=\begin{bmatrix}0\\0\\ \dfrac{\partial\psi}{\partial x}\\ \dfrac{\partial\psi}{\partial y}\end{bmatrix},\quad \frac{\partial F}{\partial t}=0,\quad \frac{\partial \psi}{\partial t}=0\,. \tag{17}$$

These identities (16) and (17) are substituted appropriately into the boundary condition (15):

$$-\left[p_1(T)\quad p_2(T)\quad p_3(T)-\nu\frac{\partial\psi(T)}{\partial x}+1\quad p_4(T)-\nu\frac{\partial\psi(T)}{\partial y}\right]\mathrm{d}\boldsymbol{s}(T)+H(T)\,\mathrm{d}T=0\,. \tag{18}$$

Both the final state and the final time T are free, i.e. different choices of the control vector $\boldsymbol{u}$ will result in different values for T and the final state vector $\boldsymbol{s}(T)$. Therefore, $\mathrm{d}T\neq 0$ and $\mathrm{d}\boldsymbol{s}(T)\neq 0$. Differentials $\mathrm{d}T$ and $\mathrm{d}\boldsymbol{s}(T)$ are also independent so that (15) yields the separate boundary conditions

$$\left.\left(\frac{\partial F}{\partial \boldsymbol{s}}+\frac{\partial \psi}{\partial \boldsymbol{s}}\nu-\boldsymbol{p}\right)\right|_T=0 \tag{19}$$

and

$$\left.\left(\frac{\partial F}{\partial t}+\frac{\partial \psi}{\partial t}\nu+H\right)\right|_T=0. \tag{20}$$

Using (16), (17) and (19) the final costate vector we get

$$\boldsymbol{p}(T)=\begin{bmatrix}0\\0\\ \nu\dfrac{\partial\psi}{\partial x}-1\\ \nu\dfrac{\partial\psi}{\partial y}\end{bmatrix} \tag{21}$$

and the Hamiltonian at time T becomes:

$$H(T)=0. \tag{22}$$

The components of the costate vector $\boldsymbol{p}(T)$ can be added to the condition (21) with respect to the Hamiltonian (10) to obtain a coefficient ν, thus

$$H(T)=p_3v_x(T)+p_4v_y(T)=\left(\nu\frac{\partial\psi}{\partial x}-1\right)v_x(T)+\nu\frac{\partial\psi}{\partial y}v_y(T)=0. \tag{23}$$

A logical consequence is:

$$\nu=\frac{v_x(T)}{v_x(T)\dfrac{\partial\psi}{\partial x}+v_y(T)\dfrac{\partial\psi}{\partial y}}. \tag{24}$$

The MATLAB® technical programming language has been used for the numerical solution of the optimization problem. This algorithm was turned into MATLAB statements:

1. The numerical solution of the Cauchy problem for the four dimensional system of nonlinear differential equations (2) with the initial condition of

$$\boldsymbol{s}_0=\left(v_{\mathrm{x}}(0),v_{\mathrm{y}}(0),x(0),y(0)\right) \tag{25}$$

respects a typical state of motion of the ski jumper system's center of mass at the start of the optimized flight. Apart from the initial condition, it is necessary to define a reference jump by tabulated functions $L = L(t)$ and $D = D(t)$ too.

2. Assessment of the flight time T, for that holds equality (6).

3. Numerical solution of the four dimensional system of nonlinear differential equations (14) with boundary conditions (21) with respect to identity (24).

4. Minimize a Hamiltonian to get an optimal time course for the control variables that facilitate the solution of the equations of motion (2) if the regression dependence of quantities L and D on control variables is known. For this purpose, it is necessary to use data from wind tunnel experiments with athletes or models of athletes positioned in accordance with real postures.

Optimization of the reference jump with three control variables

Below are the supposed aerodynamic characteristics of the model described by Seo, Watanabe and Murakami (2004) in the form of regression function

$$D=\sum_{i=0}^{4}\sum_{j=0}^{2}\sum_{k=0}^{2}a_{ijk}\alpha^i\beta^jV^k,\ L=\sum_{i=0}^{4}\sum_{j=0}^{2}\sum_{k=0}^{2}b_{ijk}\alpha^i\beta^jV^k \tag{26}$$

With the tabulated regression coefficients a_{ijk} and b_{ijk}. The control vector included three components: α, β and V. The set U^r contains at each time t the intervals in the form $\langle\alpha_0-\sigma_\alpha,\alpha_0+\sigma_\alpha\rangle$, where α_0 is a function of time describing the changing of the angle α in the reference flight (TABLE 1) and σ_α is the maximal standard deviation from the field studies made by Schmölzer and Müller (2005) – analogously for the other control variables. The values m = 65.5 kg and ρ = 1 kg·m^{-3} have been used for the all computer simulations. The initial state of motion was set to s_0 = (25.93 m·s^{-1}; 2.49 m·s^{-1}; 0 m; 0 m), because the supposed angle of the slope of the ramp was 11°, in run velocity 25.93 m·s^{-1} and take off velocity perpendicular to the ramp was 2.5 m·s^{-1} in accordance with Schmölzer and Müller (2005).

As curve S was selected an abscissa between the origin of the coordinate system O_{xy} and the position of the center of mass of the ski jumper system at the instant 5.5 s according to the equations of motion (2) solution. The trajectories of the reference and optimized jumps were compared with the K = 125 m jumping hill profile in Frenštát p. Radhoštěm (Czech Republic) and appropriate lengths of the jumps were evaluated. The projection of the jumping hill profile on the vertical plane, including the coordinate system $O_{xy,}$ was based on the measurement of the coordinates of surface points on the jumping hill by using the Leica TCR307 theodolite.

RESULTS AND DISCUSSION

The figures show the reference and optimal time dependence of the control variables (Fig. 1), aerodynamic drag and lift forces (Fig. 2), magnitude of the ski jumper system's center of mass velocity vector and it's vertical and horizontal components (Fig. 3). The optimal correction of the flight course increases the jump length from 100.7 m for the reference jump to 117.7 m – i.e. a length increment of 17% (Fig. 4). It has been

Fig. 1
Optimized and reference time variations of angles α, β and *V*

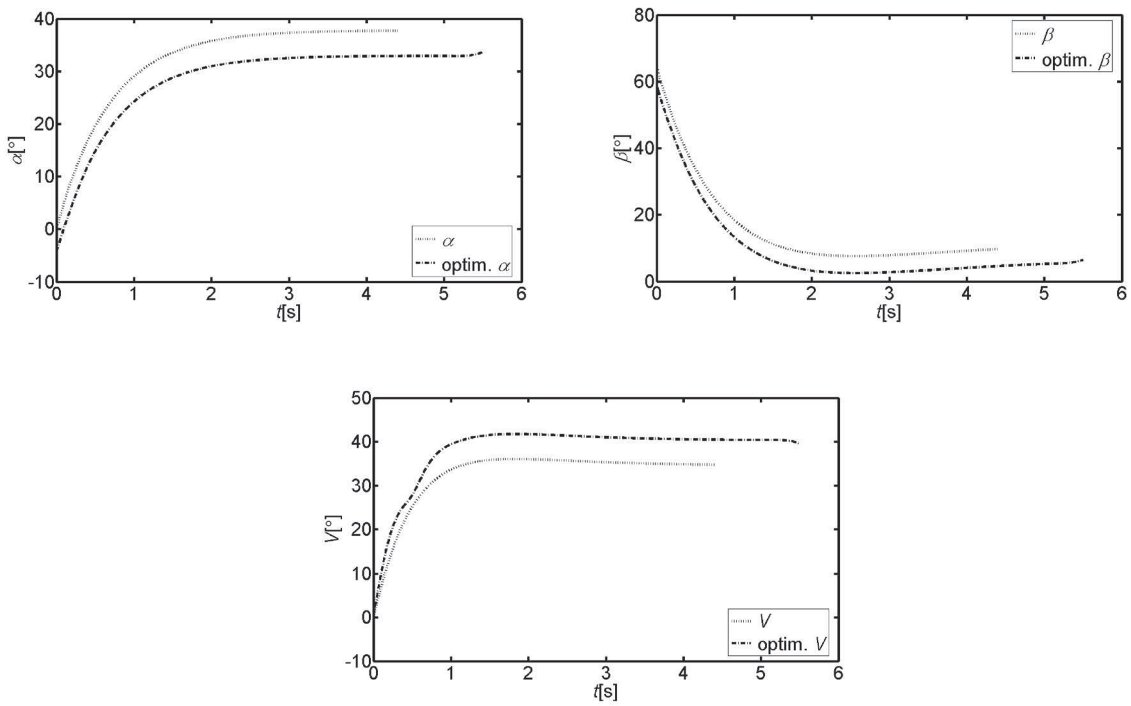


Fig. 2
Optimized and reference time variations of drag and lift forces

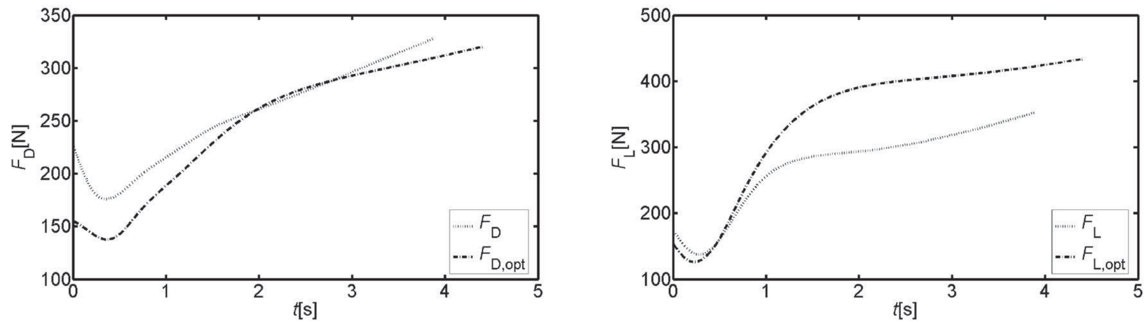


TABLE 1
The reference jump angle of attack (α), body ski angle (β) and ski opening angle (*V*) for model of the ski jumper at given times *t*. The angles correspond to the mean values found in the field. The values were selected from Schmölzer and Müller (2002)

t **[s]**	**0**	**0.2**	**0.4**	**0.7**	**1.0**	**1.2**	**1.5**	**2.0**	**4.0**	**5.5**
α_0 **[°]**	0	7	14	25	30.2	32.6	34.8	36.1	37.1	36.2
β_0 **[°]**	63	49	43	26	16.4	13	10.4	10.3	10.8	9.3
V_0 **[°]**	0	13	20	31	35	35	35	35	35	35

Fig. 3
Optimized and reference time variations of the magnitude, vertical and horizontal components of the ski jumper's center of mass velocity vector

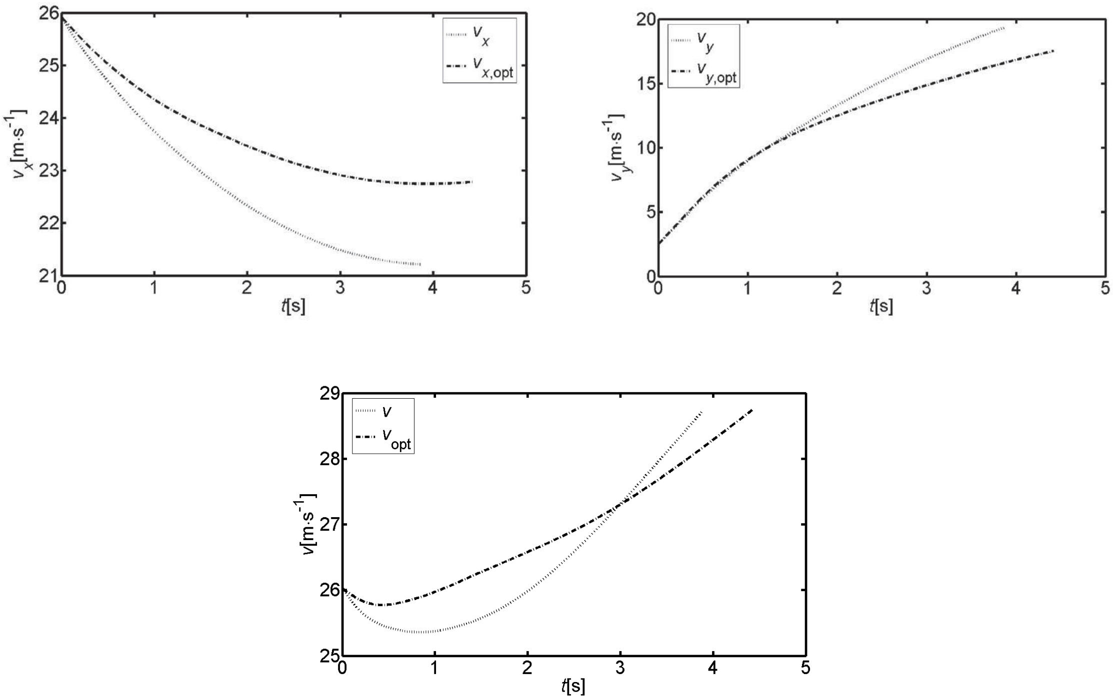
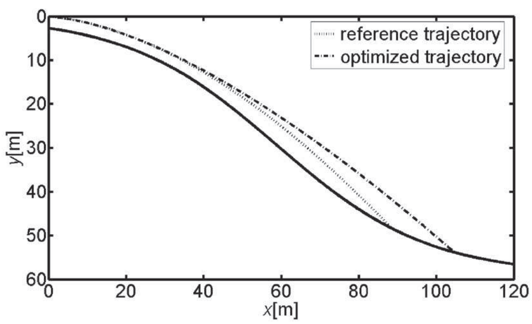


Fig. 4
Flight path for the reference and optimized jump execution in the case of Frenštát p. R. Jumping Hill (K = = 125 m)



discovered that a ski jumper should minimize angles α and β (Jošt, Kugovnik, Strojnik, & Colja, 1997; Seo, Murakami, & Yoshida, 2004; Virmavirta et al., 2005), the remaining angle *V* should, however, be maximized for some time in the interval from 0 s to 0.5 s, while the optimal angle *V* should gradually approach the upper bound of the set U^r, containing the acceptable values of angle *V*. As found by Seo, Murakami and Yoshida (2004), the ski opening angle should be increased in the first half of the flight, and than kept at a constant value.

In terms of aerodynamic forces, it is particularly optimal to minimize the drag force within 0.5 s, while at 2–3 s, the value of the drag approaches the value corresponding to the reference jump, later again becoming a strong requirement to minimize the drag force. The best option is to minimize the lift force up to 0.5 s and to subsequently maximize (Fig. 2). In contrast to the study of Schmölzer and Müller (2005), Fig. 2 shows a local minimum of the aerodynamic drag and lift forces.

The difference between the x-th resp. the y-th components of the velocity vector gradually increases and, for example, at the moment 3 s extends to $v_{x,\mathrm{opt}} - v_x = 1.5\ \mathrm{m \cdot s^{-1}}$ resp. $v_{y,\mathrm{opt}} - v_y = -2.5\ \mathrm{m \cdot s^{-1}}$. The relative differences in the y-th component of the velocity vector are remarkably bigger that in the x-th component. Initially it is $v_{\mathrm{opt}} > v$, afterwards $v_{\mathrm{opt}} < v$. This changeover is due to the new directive to maximize the lift with some delay, similarly as in the increasing influence of the optimization to the vertical component of the velocity vector. According to Jošt, Kugovnik, Strojnik and Colja (1997) in order for the jump to be performed successfully, it is right to maximise the resultant speed in the first part of the flight and to maximize the x-th component of the velocity vector during the entire time of the flight, unlike shown in Fig. 3, in particular in the first part of the flight.

Meile et al. (2006) stated that current computer fluid dynamics tools do not seem to be capable of simulating the aerodynamic forces acting on the ski jumper system. Thus, the experimental investigation of the aerodynamic forces remains, so far, essential.

The ways of how to achieve the optimal time course of flight position angles can distinctly differ depending on the athlete – compare the gold medallist Amman and the silver medallist Malysz (Schmölzer & Müller, 2005). This means that the reliability of the optimization studies could be improved by having data for a deeper insight into the aerodynamics of the individual athlete. There exist several strategies of how to solve the changes at the angular momentum at the early flight phase (Hildebrand, Drenk, & Müller, 2007).

CONCLUSION

The solution of the optimization problem with three control variables allowed for the correction of the reference flight style to maximize flight distance with respect to control limits on control variable values. Unambiguous directives to minimize the angle of attack and body angle relative to the skis correspond to the requirement to minimize the aerodynamic drag primarily in the first part of the flight (Fig. 1). From the viewpoint of aerodynamics, it is interesting to note that, during the flight, gaining grain aerodynamic lift in this sense that is becoming majoritarian, that is, to maximize lift demand above all by increasing the ski opening angle compared with the reference values (Fig. 1). Even in the middle part of the flight, the optimal aerodynamic drag force is slightly higher than the reference drag as can be seen in Fig. 2. In terms of kinematics, the trajectory of the ski jumper system's center of mass is straightening compared to the reference trajectory approximately at a distance 60 m from the jumping hill edge, where the difference of the height above the jumping hill exceeds 0.5 m.

If, in the future, correct work contrary to the presented study maps the flight style of the exact ski jumper, data collection might involve the following steps:

- mathematical modeling of the selected jumping hill profile, for example on the basis of topography measurement,
- the setting of admissible intervals of control variables that respect the ski jumper's mental and motor abilities,
- determination of the dependence of aerodynamic drag and lift forces acting on the ski jumper system on the flight via regression analysis of experimental data.

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REFERENCES

Grozin, E. A. (1971). *Ski jumping*. Moscow: Fizkultura i sport.

Hermsdorf, H., Hildebrand, F., Hofmann, N., & Müller, S. (2008). JUMPICUS – computer simulation in ski jumping. In M. Estivalet & P. Brisson (Eds.), *The Engineering of Sport 7* (pp. 491–497). Paris: Springer-Verlag.

Hildebrand, F., Drenk, V., & Müller, S. (2007). Stability during ski jumping flight phase. In E. Müller, S. Lindinger, T. Stöggl, & V. Fastenbauer (Eds.), *Abstract Book of the 4th International Congress on Science and Skiing* (pp. 106). Salzburg: University of Salzburg.

Jin, H., Shimizu, S., Watanuki, T., Kubota, H., & Kobayashi, K. (1995). Desirable gliding styles and techniques in ski jumping. *Journal of Applied Biomechanics, 11*, 460–474.

Jošt, B., Kugovnik, O., Strojnik, V., & Colja, I. (1997). Analysis of kinematic variables and their relationship to the performance of ski jumpers at the world championship in ski flight at Planica in 1994. *Kinesiology, 29(1)*, 35–44.

Lewis, F. L., & Syrmos, V. L. (1995). *Optimal control*. New York, NY: John Wiley & Sons.

Meile, W., Reisenberger, E., Mayer, M., Schmölzer, B., Müller, W., & Brenn, G. (2006). *Experiments in Fluids, 41(6)*, 949–964.

Müller, W., Gröschl, W., Müller, R., & Sudi, K. (2006). Underweight in ski jumping: The solution of the problem. *International Journal of Sports Medicine, 27(11)*, 926–934.

Müller, W., Platzer, D., & Schmölzer, B. (1996). Dynamics of human flight on skis: Improvements in safety and fairness in ski jumping. *Journal of Biomechanics, 29(8)*, 1061–1068.

Murakami, M., Hirai, N., Seo, K., & Ohgi, Y. (2008). Aerodynamic study of ski jumping flight based on high speed video image. In M. Estivalet & P. Brisson (Eds.), *The Engineering of Sport 7* (pp. 449–456). Paris: Springer-Verlag.

Pierre, D. A. (1969). *Optimization theory with applications*. New York, NY: John Wiley & Sons.

Pontryagin, L. S., Boltyanskii, V. G., Gamkrelidze, R. V., & Mischenko, E. F. (1962). *The mathematical theory of optimal processes*. New York, NY: Wiley Interscience.

Remizov, L. P. (1984). Biomechanics of optimal flight in ski jumping. *Journal of Biomechanics, 17(3)*, 167–171.

Schindelwig, K., & Nachbauer, W. (2007). Evaluation of a simple ski jumping model. In E. Müller, S. Lindinger, T. Stöggl, & V. Fastenbauer (Eds.), *Abstract Book of the 4th International Congress on Science and Skiing* (pp. 105). Salzburg: University of Salzburg.

Schmölzer, B., & Müller, W. (2002). The importance of being light: Aerodynamic forces and weight in ski jumping. *Journal of Biomechanics, 35(8)*, 1059–1069.

Schmölzer, B., & Müller, W. (2005). Individual flight styles in ski jumping: Results obtained during Olympic Games competitions. *Journal of Biomechanics, 38(5)*, 1055–1065.

Seo, K., & Murakami, M. (2003). The optimization study of ski jumping flight on the basis of aerodynamic data. *Descente Sports Science, 24*, 31–37.

Seo, K., Murakami, M., & Yoshida, K. (2004). Optimal flight technique for V style ski jumping. *Sports Engineering, 7(2)*, 97–103.

Seo, K., Watanabe, I., & Murakami, M. (2004). Aerodynamics force data for a V style ski jumping flight. *Sports Engineering, 7(1)*, 97–104.

Vaverka, F. (1994). Somatic problems associated with the flight phase in ski jumping. *Studia monografia AWF we Wroclawiu, 40*, 123–128.

Virmavirta, M., Isolehto, J., Komi, P., Brüggemann, G. P., Müller, E., & Schwameder, H. (2005). Characteristics of the early flight phase in the Olympic ski jumping competition. *Journal of Biomechanics, 38(11)*, 2157–2163.

Víteček, A. (2002). *Optimální systémy řízení* [Vysokoškolské skriptum]. Ostrava: Vysoká škola báňská – Technická univerzita, Fakulta strojní.

PONTRJAGINŮV PRINCIP MAXIMA A OPTIMALIZACE STYLU LETU VE SKOKU NA LYŽÍCH
(Souhrn anglického textu)

Existuje několik faktorů (počáteční poloha skokana na lyžích a její změny v průběhu přechodové fáze letu, velikost a směr vektoru rychlosti pohybu těžiště skokana, velikost aerodynamické odporové a vztlakové síly apod.), které určují trajektorii soustavy skokan + lyže a tím i dosaženou délku skoku. Cílem studie je představit metodu řešení úlohy optimálního řízení letové fáze skoku na lyžích se třemi omezenými řídicími proměnnými – úhel náběhu (α), úhel trup vs. lyže (β), úhel levá lyže vs. pravá lyže (V) – na základě Pontrjaginova principu maxima. Kritériem optimality byla zvolena délka skoku. Jako zdroj informací o závislosti veličin L (lift area) a D (drag area) na řídicích proměnných byla použita převzatá regresní funkce s tabelovanými regresními koeficienty. Srovnány byly trajektorie referenčního a optimalizovaného skoku s profilem můstku K = 125 m ve Frenštátě pod Radhoštěm a stanoveny odpovídající délky skoku, aerodynamické odporové a vztlakové síly, velikosti rychlosti pohybu těžiště soustavy skokan + lyže, její vertikální a horizontální složky. Aby byly respektovány reálné polohy v letové fázi skoku, přípustné hodnoty řídicích proměnných náležely v každém okamžiku ohraničené množině. Bylo zjištěno, že skokan by měl na ohraničené množině přípustných hodnot řídicích proměnných minimalizovat úhly α a β, úhel V naopak maximalizovat. Prodloužení skoku vlivem optimalizace je 17 %. Pro možnost dalšího výzkumu je nezbytné využití regresní analýzy pro experimentální data při určení závislosti aerodynamických sil působících během letu na soustavu skokan + lyže. To platí také pro aplikaci kontrolních proměnných vztahujících se k základním fyzickým a psychickým vlastnostem skokanů na lyžích.

Klíčová slova: počítačová simulace, skokan na lyžích, optimální řízení, aerodynamické síly.

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First-line publications

Uhlář, R. (2006). Dynamika letové fáze skoku na lyžích: Critical review. *Česká kinantropologie, 10(2)*, 71–86.

Uhlář, R., & Janura, M. (2007). Ski jump flight as an optimization task. In V. Linnamo, P. V. Komi, & E. Müller (Eds.), *Science and Nordic Skiing* (pp. 237–245). Oxford: Meyer & Meyer Sport.

Uhlář, R., & Janura, M. (2009). Simulace letové fáze skoku na lyžích a vliv vybraných parametrů na délku skoku. *Studia Sportiva, 3(2)*, 5–13.